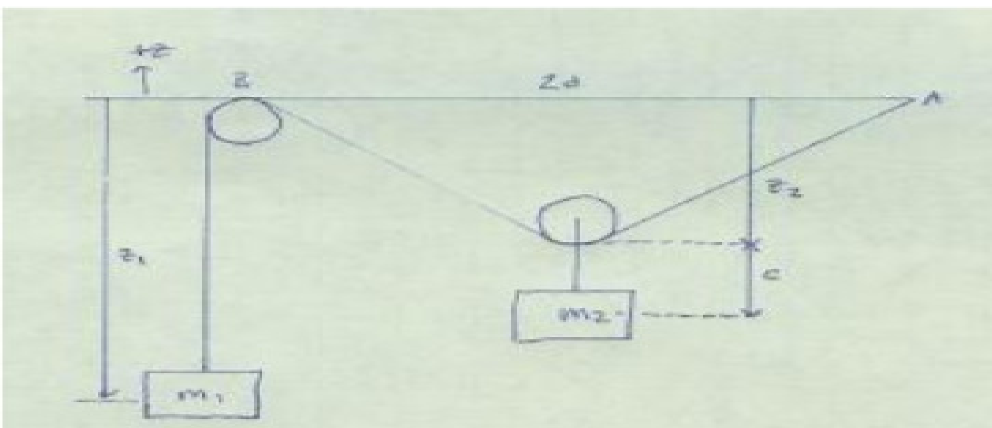
ISI – Bangalore Center – B Math - Physics I – Mid Semester Exam
Date: 28 February 2019. Duration of Exam: 3 hours
Total marks: 70

Q1. [Total Marks: 7+4+4 =15]

Consider the system of pulleys and masses shown in the following figure. A rope of fixed length *b*, is fixed at point A, and runs over a fixed pulley at point B a distance *2d* away. The mass m_1 is attached to the end of the rope below point B, while the mass m_2 is held on to the rope by a pulley between A and B. Assume the pulleys are massless and have zero size. The potential energy function of the following system is given by

$U = mgz_1 + mg(z_2 - c)$ where z_1, z_2 are both negative and measured perpendicularly from the line AB.



a.) Show that the system has a point of equilibrium only if $\frac{b}{d} > f(m_1, m_2)$ and $\frac{m_1}{m_2} > \frac{1}{2}$. Determine the function *f*.

b.) Show that the point of equilibrium is stable.

c.) Describe qualitatively the motion of the system under small perturbations around the stable equilibrium.

Q2. [Total Marks: 9+5+6=20]

A damped harmonic oscillator satisfies the equation $\ddot{x} + 2K\dot{x} + \Omega^2 x = 0$.

a.) At time t=0, the particle is projected from the origin towards the positive x axis with speed u. If $K > \Omega$, find x(t) in the subsequent motion and show that the particle will never reach the origin in finite time.

b.)If $K > \Omega$, what is the time t_0 required for the particle to come to a momentary rest after being projected? Express your answer as $Kt_0 = f(x)$ where $x = \frac{\delta}{K}, \delta = \sqrt{(K^2 - \Omega^2)}$.

Determine the function $f(x)$.

c.) From the above, determine the limitng value of t_0 when the system is just overdamped (i.e. overdamped but very close to critical damping) and compare it to the corresponding value when the system is extremely overdamped $K >> \Omega$.

Can you draw any conclusion whether with the given initial conditions as above a critically damped motion would come to momentary rest faster or slower than the overdamped case?

Q3. [Total Marks:4+5+2+4+5 =20]

Particle 1 of mass m is connected with two other identical particles (particle 2 and particle 3) with massless rigid rods so that the distances between particles 1and 2 and between particles 1 and 3 are fixed. You can assume that the distance between particle 1 and 2 is l and between 1 and 3 is also l. The three particles are on a frictionless horizontal table. The system of masses can move freely on the table subject to the constraint described.

a.) List all the forces of constraint.

b.) Show that the forces of constraints do not do any work.

c.) How many independent coordinates are required to describe this system?

d.) Write the Lagrangian of this system in terms of any set of independent coordinates of your choice.

e.) Name three physical quantities that you expect to be conserved in the evolution of this system given any set of initial conditions. Are these conservation laws evident from the Lagrangian? Explain why or why not?

Q4. [Total Marks: 4+4+4+3=15]

Two stars S_1 and S_2 of mass *m* and *2m*, forming a binary star pair move under their mutual gravitational interaction force .

a.) Starting with the equation of motion of each particle, show that the equation of motion for the relative coordinate vector is given by $\ddot{\vec{r}} = \frac{3G}{r^2}\hat{r}$

b.) Suppose the orbit of S_1 relative to S_2 is an ellipse with S_2 at one focus, show that the orbits of S_1 and S_2 in the zero momentum frame are similar ellipses, each with its focus at C, where C is the centre-of-mass

c.) For the system described in part b.) determine the ratio of the major axes of their orbits in the zero momentum frame and determine if they have the same time period.

d.) Now consider the sun-earth system with the sun being essentially fixed because of its huge mass compared to that to the earth. Assume that initially the earth's orbit is circular. If the mass of the sun (still huge) suddenly decreases by half, what orbit will the earth then have ? Will the earth escape the solar system ? Justify your answer.